Algebraic and Graphical Solutions of Linear Equations

Steps for Solving Linear Equations with One Variable

Step 1: **EXPAND** - Remove any parentheses using the Distributive Property

Step 2: **COLLECT** - Combine like terms on each side of the equation

Step 3: **ADD** - Use the Addition Property of Equality to get all variables on one side and all constants on the other side.

Step 4: **Multiply** - Use the Multiplication Property of Equality to get the coefficient of the variable to equal 1.

Step 5: **Check** - Check the solution to verify that it satisfies the original equation.

Remarks

In step 3 we add/subtract to/from each side

In step 5 we multiply/divide each term

If fractions are present, we can multiply each term by the LCD before solving

If decimals are present, we can multiply each term by 10, 100, etc. to remove the decimals before solving
Example

\[ -\frac{14}{5} \cdot \frac{3(x-16)}{20} \]

**LCD = 20**

\[-56 = 3(x-16)\]

**Expand**

\[-56 = 3x - 48\]

**Collect**

There is nothing to collect

**Add 48**

\[-8 = 3x\]


Example

\[ -\frac{14}{5} \cdot \frac{3(x-16)}{20} \]

**Divide by 3**

\[-\frac{8}{3} = x\]

**Check**

\[\frac{3}{20} \left( -\frac{8}{3} - 16 \right) = \frac{3}{20} \left( -\frac{56}{3} \right) = \frac{-56}{20} = -\frac{14}{5}\]

Remarks

When only two terms are present, we can cross-
multiply instead of using the LCD

\[
\frac{a}{b} \cdot \frac{c}{d} \Rightarrow ad = bc
\]

\[-\frac{14}{5} = \frac{3(x-16)}{20} \Rightarrow -14 \cdot 20 = 5 \cdot 3(x-16)\]

When we check, we may begin with the harder side
Example

\[ \frac{5}{6} (12x + 6) - 4x + 5 = -20 \]

Expand

\[ \frac{5}{6} \left( \frac{12x}{1} + \frac{6}{1} \right) - 4x + 5 = -20 \]
\[ \frac{5}{6} \cdot 2x + \frac{5}{6} \cdot 1 - 4x + 5 = -20 \]
\[ 10x + 5 - 4x + 5 = -20 \]

Collect

\[ 6x + 10 = -20 \]

Add 10

\[ 6x = -30 \]

Example

\[ \frac{5}{6} (12x + 6) - 4x + 5 = -20 \]

Divide 6

\[ x = \frac{-30}{6} = -5 \]

Check

\[ \frac{5}{6} \left( 12(-5) + 6 \right) - 4(-5) + 5 \]
\[ \frac{5}{6} \left( -60 + 6 \right) + 20 + 5 \]
\[ \frac{5}{6} \left( -54 \right) + 25 \]
\[ -45 + 25 \]
\[ -20 \]

Example

\[ \frac{3y}{5} = \frac{5t - 2}{3} + 5 \]

\[ \frac{3y}{5} - \frac{3}{3} = \frac{5t - 2}{3} + \frac{2}{1} \]

\[ 15 \cdot \frac{3y}{5} - 15 \cdot \frac{3}{3} = 15 \cdot \frac{5t - 2}{3} + 15 \cdot \frac{2}{1} \]

\[ 9t = 5(5t - 2) + 75 \]

\[ 9t = 25t - 10 + 75 \]

\[ -65 = 16t \]

\[ \frac{65}{16} = t \]

\[ \left\{ \begin{array}{l}
\frac{65}{16} \\
\frac{-65}{16}
\end{array} \right. \]
Example

Intersection Method

\[
\frac{3t}{5} - 3 = \frac{5t - 2}{3} + 2
\]

\(Y_1 = \text{Left Hand Side}\)
\(Y_2 = \text{Right Hand Side}\)

\[
\text{Plot} \quad \text{Plot} \quad \text{Plot}
\]

Example

Zero Method

\[
\frac{3t}{5} - 3 = \frac{5t - 2}{3} + 2
\]

\(Y_1 = (\text{Left Hand Side}) \, - \, (\text{Right Hand Side})\)

\[
\text{Plot} \quad \text{Plot} \quad \text{Plot}
\]

Remarks

Finding zeros of a linear function \(y = f(x)\) with two variables is exactly equivalent to solving a linear equation \(f(x) = 0\) with one variable.

When we are asked to find the zeros for a function \(y = f(x)\) we are being asked to solve the equation \(f(x) = 0\).
Zero of a Function

Any number \( a \) for which \( f(a) = 0 \) is called a zero of the function \( y = f(x) \).

If \( a \) is real, then \( a \) is an \( x \)-intercept of the graph of the function.

Remarks

The following are equivalent:

- An \( x \)-intercept of \( y = f(x) \)
- A real zero of \( y = f(x) \)
- A real solution to \( f(x) = 0 \)

Example \( y = 2x + 5 \)

The \( x \)-intercept is \(-\frac{5}{2}\)

The real zero is \(-\frac{5}{2}\)

The real solution to \( 2x + 5 = 0 \) is \(-\frac{5}{2}\)

Solving a Linear Equation with a Graphing Calculator

We have two methods.

The hardest part of this method is finding a suitable viewing window.

We can use the Table, and try different \( x \) values to estimate the \( x_{min} \), \( x_{max} \), \( y_{min} \), and \( y_{max} \).
Solving a Linear Equation with a Graphing Calculator

Method 1 – Intersection Method

1. Graph the left hand side of the equation into y1
2. Graph the right hand side into y2
3. Find the points of intersection

Example

Identify each side

\[ \frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4} \]

Check table to determine viewing window
Example

\[ \frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4} \]

Select 2nd curve

Set lower bound (left side of intersection)

Set upper bound (right side of intersection)
Example

\[ \frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4} \]

Guess

Our solution is \( x = -2 \)

Example

\[ \frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4} \]

Intersection – Solution is the \( x \)-value

Our solution is \( x = -2 \)

Solving a Linear Equation with a Graphing Calculator

Method 2 – \textbf{x-intercept Method}

1. Graph the left hand side minus the right hand side into \( y1 \)
2. Find the \( x \)-intercepts
Example

Identify each side

Left hand side
LHS = \( \frac{3(4x + 32)}{4} \)

Right hand side
RHS = \( \frac{3(20 - 2x)}{4} \)

Set viewing window

Since we are finding the zero (x-intercept) we want to be close to the x-axis.

Example

Set viewing window

TI-89 F5  TI-84 2nd Calc
Example

\[
\frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4}
\]

Lower Bound (left side of x-intercept)

\[
x = -2.10884, y = 4.53782
\]

Upper Bound (right side of x-intercept)

\[
x = -1.93275, y = 3.02521
\]

TI 84 Guess – anywhere between bounds

Our solution is \(x = -2\)
Solve Using TI-89

We use the solve feature of the TI-89 from the home screen

\[ \text{Solve( Equation 1 , x) } \]

or \( y \)

Under F2

Must be an equation

Example

\[
\frac{3(4x + 32)}{4} = \frac{3(20 - 2x)}{4}
\]

Our solution is \( x = -2 \)